Variations in Position of Columns and Slabs

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ABSTRACT

Design details often lack tolerances, causing problems in construction. Though standards and codes may supply them, tolerances are uncoordinated from trade to trade, and are usually neither mandatory nor enforceable. Designers should allow for them, but to do so rationally, must know what variations occur in practice. This information is often not available. What is to be done to supply it?

This paper describes measurements on 4 buildings, three recently constructed, and one around 20 years old, of differences in cast-in-place (CIP) concrete slab edge position, in CIP floor to floor dimensions, in spacing of precast concrete (PC) columns, in spacing of steel structural members, and in spacing of CIP columns.

Measurements that were evidently intended to be identical were grouped into sample datasets for statistical analysis. Tolerances required in details to avoid chipping, patching, and other ad hoc fixes for all but extreme cases (defined by confidence intervals based on estimated population standard deviations) were determined for the corresponding populations.

Even building on the existing literature of construction variations, these limited measurements clearly cannot fully supply the rational foundation needed for assigning tolerances, but a start has to be made somewhere. The paper offers guidance on reasonable approaches to take, even in the face of missing information. However, the primary objective is to highlight the urgency, as well as the benefits, of investigating variations in the sizes and positions of building components.

INTRODUCTION

As builders who lay out components per designer's details might put it, mistakes are inevitable. But the word mistake is not always appropriate. All measurement entails a degree of uncertainty, so variation must be expected and accommodated. When size and position deviate more than the tolerance allowed for by the details, the builder has to make adjustments on the spot, often to the detriment of performance.

In current practice, designers rarely make explicit provision for variation when planning details of construction. Without considering the implications in detail, they rely on standards to specify tolerances and on builders to “do the cutting and remedial work required to make the several parts of the Work come together properly”[1]. The need for such cutting and fitting may be thought of as the result of carelessness or error—a failure to properly coordinate the various parts—but if one looks into the unavoidable uncertainties, and the tolerances specified in standards, it becomes evident that problems can be inherent in details that make no provision for tolerance or adjustment. In contrast, contractors, fabricators, and trade associations regard the coordination of tolerances to be the responsibility of the designer.

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[1] CCDC2, GC3.13.1

Illustration 1: Walls and supporting slab misaligned; on one side the slab projects from the wall sheathing, on the other it's retracted. The same detail applies to both—the designer envisioned the slab edge and face of sheathing in alignment.

Building Envelope Engineering

Failure to provide for variations in position can have serious consequences. In cavity wall construction, the air space is often built larger or smaller than its design width, as a result of positioning of cladding, structure, and insulation. Air spaces reduced to nil are not unknown, and sometimes insulation is left off to make room. Bridging of the cavity by exterior moisture, poor drying, and excess heat loss are possible results of not including tolerances in design of details. Structural supports for cladding, like ledger angles, if not provided with adequate adjustment often project too far—resulting in cutting on site—or not far enough—resulting in inadequate support of the cladding. The *Summary Report* of the ASCC inter-industry meeting provides many examples of things that commonly go wrong, and concludes that because tolerances for different industries are not coordinated, that designers should be responsible for coordination of tolerances.[2]

Latta, in CDB 179, identified tolerances as a problem in the construction industry, and set out the accuracy of measurement possible with common construction measuring tools. However, he also observed, based on the work of ACI Committee 117 (1962 measurements of CIP column spacings), and the NRC/DBR (precast panel joint widths measured on 6 stories of a 30 story building), that variations in actual construction were much larger. For concrete columns, a typical variation from intended position was 25 mm, with a maximum of 50 mm. The nominal 9.5 mm joints between precast panels varied from 0 to 19 mm. His conclusion is as relevant today as it was in 1975:

> "In general, it is probably reasonable to say that the standard of accuracy thought to be attainable in building construction is often much higher than that actually attained in practice. Such a statement is not intended to be a criticism of the building industry. It implies, rather, the need for designers, specifiers and constructors to be more realistic about the situation. Designers may have to accept the present standard of inaccuracy as a fact of life and design all facets of the building in such a way that errors can be accommodated in construction without expensive and time-consuming remedial measures on site. A design that demands higher accuracy than can normally be attained will inevitably be more troublesome to build. Specifiers should not call for unnecessarily tight tolerances; they will only lead to an excessive number of rejections or to protracted arguments. In turn, constructors should review their present practices with a view to minimizing inaccuracies. If only the gross errors were eliminated, or even if they were detected at an early stage, many a job would run more smoothly."

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[2] ASCC, January 2006, report of a meeting co-sponsored by the American Architectural Manufacturers Association, American Concrete Institute, American Institute of Steel Construction, American Society of Civil Engineers, Construction Institute, Concrete Reinforcing Steel Institute, Floor Covering Installation Contractors Association, Portland Cement Association, Post-Tensioning Institute, and the Strategic Development Council of the ACI Concrete Research and Education Foundation.

TOLERANCES IN DESIGN

CMHC's *Construction Tolerances* article provides an example of tolerances worked out in a simple (and conservative) way for masonry veneer steel stud wall details, and discusses problems with the applicable standards, but provides little basis for the designer's choice of what tolerances to specify instead. Study of the standards for various types of material and construction will reveal that allowing for the tolerances they specify may not be adequate. Specified tolerances often conflict, between construction types, and even within a given type. ACI standards allow variation in formwork dimensions and in rebar placement, while specifying tolerances for minimum cover such that cover over the rebar can be forced out of tolerance by variations in form dimensions and rebar location that are both within tolerance. Tolerances for manufacture of hot-rolled structural shapes until recently allowed skew, bowing, and other variations in dimensions and geometry of individual members that would preclude achievement of the tolerances specified for erection. Canadian concrete standards suggest tolerances for variance, but this makes rejection of any individual departure impossible.

Ballast provides a compilation of standardized tolerances for many materials, drawn from North American standards, and works out suggested methods of coordination for a number of details, but he takes the specified tolerances as given. He notes that the standardized tolerances have evolved over the last century, and embody much practical experience, but also that each industry has a tendency (and who can blame them) to write their tolerances to suit narrow interests, with the effect of externalizing, but not solving, possible problems.

Compared to the single example provided by CMHC and Ballast's several examples, Milberg and Tommelein offer a comprehensive framework for coordinating tolerances in assemblies, based on procedures commonly used in manufacturing. Before their scheme can be applied, however, the designer must choose tolerances for each element. Their scheme uses absolute tolerances, although they say it could be extended to incorporate a statistical approach.

First published from 1979 to1989, ISO standards 3443.1 to 3443.8 set out an approach to coordinating tolerances of components in an assembly, and specifying tolerances, that is statistically rigorous. It could possibly be melded with the Milberg and Tommelein approach. Parts 3 and 4 should suffice for a designer to coordinate tolerances in designing details, although the whole series, plus ISO 1803-1, would be required to cover the subject from definitions to verification of performance at the job site. Still, in order to apply ISO procedures, a designer must know or guess the standard deviations of error in manufacture, setting out, and erection, of each component in a detail, and be able to decide what rates of rejection are economically acceptable.

None of the foregoing offers the designer a reliable way to decide what tolerances to use, before he attempts to coordinate them. The question—how much variation occurs in practice?—remains unanswered.

TOLERANCES IN PRACTICE

Begel and Bohnhoff's systematic reporting of measured variation in pole-frame agricultural buildings is a welcome exception to the casual anecdotes usually found in the literature. They made extensive measurements of existing buildings, and Bohnhoff subsequently published a suggested standard for pole-frame construction based on the variation they observed. They made an important point:

> “Establishment of a document that includes recommended construction tolerances requires careful research. First, decisions must be made as to what tolerances to include in such a document. Second, the type and magnitude of each tolerance must be carefully selected. Before the latter can be accomplished, information regarding the accuracy of current post-frame construction must be ascertained. Publishing guidelines that are too restrictive would

> unnecessarily increase construction costs. Conversely, introducing a document with tolerances that are much more lenient than those found in current practice would do little to change current practices.”

The data reported here is not extensive enough to definitively answer the question, but it does serve as a starting point—and suggests standard deviations that should be treated as minimums for the construction types surveyed. Three of the four buildings measured are of recent construction, and one of these was designed and built for a demanding and sophisticated owner by some of the largest and most capable design and construction firms in Canada.

METHODOLOGY

The methods of measurement, and elements measured varied. Building 1 was a recently constructed residential building with exposed CIP balcony slabs. A Leica Geosystems HDS3000 3D laser scanner was used to capture two point clouds. Cloud-to-cloud alignment in software was used to merge the data. Least-squares fits to the point data from visible slab edges and soffits defined a plane for each visible balcony slab surface, and the geometry of the fitted planes was exported to a DXF file using the vertical intersection between two large walls to orient the vertical coordinate. Sub-sets of points were selected that, assuming the balconies were intended to be aligned, ought to have been connected by vertical lines. For each such vertical stack of points on a stack of balconies, linear regression determined if the best fit was parallel to the selected vertical axis, and indicated that it was—typically within one mm. From the point locations, offsets between adjacent floors were determined and all the offsets were grouped into a single dataset for analysis.

The manufacturer's specified standard deviation (represented by sigma, σ) for points is 6 mm at a distance of 50 m with this scanner (it would vary with angle of incidence and distance). However, if the assumption that slab edges and soffits are plane surfaces is accepted, the standard deviation of measurement is reduced, typically to 2 mm.

Illustration 4: Building 1: Photograph of balconies; outline shows extent of Illustration 3.

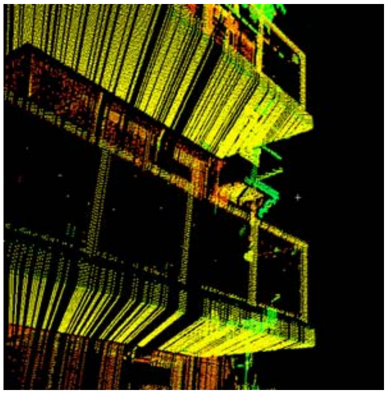

Illustration 2: Building 1: Detail of point cloud; colours indicate strength of return signal.

Point Geomatics

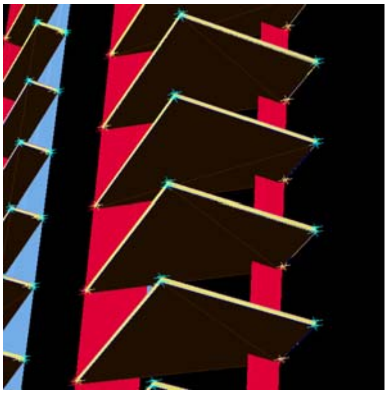

Illustration 3: Building 1: 3D Geometry of fitted planes and resulting vertices.

Building 2 was a precast concrete (PC) parkade. Spacings between faces of columns were measured with a Leica Disto 5a laser distance meter, and the data were grouped into sets of column pairs that were evidently intended to be identically spaced. The manufacturer states the accuracy of this device to be ± 1.5 mm. When it was fixed relative to a target, repeated measurements either reported the same value every time, or alternated by 1 mm between adjacent values. In short, the displayed measurements did not have enough resolution to allow characterization of the accuracy. In field conditions, there was more variation in some cases, presumably due to unsteady positioning, target reflectance, interfering reflectance from specular surfaces near the line of sight, surface roughness, and other factors. However, for Building 4, repeated measurements of the space between a pair of columns, repositioning the meter each time —10 measurements over an interval of ~5.8 m—were one of two adjacent values, 1 mm apart. Not all the reported measurements are this reliable, as can be seen from the repetitions (also in Building 4) reported in Table 1. [3]

In addition to distances between columns, column dimensions were measured with an ordinary steel tape.

Illustration 5: Building 2: Panoramic view of interior.

Building 3 was an underground CIP parkade in a recently constructed high-rise residential building. The laser distance meter was used to measure column spacings, and data were grouped in the same manner as for Building 2.

Illustration 6: Building 3: Panoramic view of interior.

Building 4 was a recently constructed CIP frame laboratory building that was unfinished on the interior. It had interstitial ceiling spaces with floors of CIP topping on metal deck in a structural steel frame suspended on HSS hangers supported by the CIP floor above. From the ceiling space, it was possible to access the gap between the exterior aluminum curtain wall and the CIP concrete floor slab.

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[3] Using a temperature-corrected steel tape and a reflectorless tacheometer, Hocking found that the accuracy of his Disto Classic 5a for short periodic measurements was ± 0.46 mm, and for additive constant measurements was ± (0.47 mm + 0.94 ppm). He experienced larger variations when making measurements in field conditions, with ranges of 15 mm in repeated measurements of diagonal dimensions of rooms. (With the tacheometer, his ranges for repeated measurements were also 15 mm).

Illustration 7: Building 4; Panoramic view of ceiling space, measurement in progress along curtain wall in background.

Adjacent to the CIP slab edge and curtain wall, a laser was set up to project a fixed horizontal reference line more-or-less parallel to the wall. Although stability was necessary, the accuracy of the reference line is not relevant, because the analysis of the resulting data looks only at variation from least-squares-fitted lines, and does not address whether the slab was horizontal, or if the wall was at the intended location and alignment.

Spacings between HSS hangers, spacings between columns, and dimensions of CIP columns were also measured, using the laser distance meter for spaces between members, and a steel tape for column dimensions.

ANALYSIS

For analysis, data were grouped into sets of dimensions that one could reasonably infer were intended to be identical, for instance:

1. spacings between seemingly aligned columns across driving aisles in parkades,
2. edges of balconies that appeared to be vertically aligned,
3. horizontal spacings between vertical structural members that appeared to be aligned in regular arrays, and that appeared to have been intended to be the same size.

Chi-square (X^2) tests of fit to the Normal distribution were done for each of the datasets. Where values of greater than 5% are reported it is reasonable to treat those datasets as representative samples from Normally distributed populations, at least for larger values of *N*. Frequency distributions (histograms) of samples for which X^2 is less than 5% (italicized), are skewed with the smallest or largest readings typically being widely separated from the rest. Data sets with extreme outliers ($X^2 < 0.1\%$) have no X^2 value entered.

RESULTS

Dataset	***Mean***	***N***	***Range***	σ	χ^2
Building 1:					
Horizontal offsets between vertically aligned CIP slab edges	-0.50 mm	262	44 mm	7.4 mm	7.6%
Vertical distance from floor to floor	2.973 m	275	77 mm	10.8 mm	10.6%
Building 2:					
Variations in PC column face dimensions about their mean values (10+18+9=37)	401,601, 798 mm	37	9 mm	1.8 mm	43%
North-to-South spaces between PC column faces, with 3 outliers removed (see Appendix)	16.788 m	41	43 mm	9.8 mm	29%
North-to-South spaces between PC column faces, with 1 extreme outlier removed (see Appendix)	16.791 m	43	145 mm	20.0 mm	*0.6%*
East-to-West spaces between PC column faces (narrow set)	6.698 m	20	42 mm	10.1 mm	51%
East-to-West spaces between PC column faces (wide set) (although χ^2 is 62%, *N* is too small)	6.891 m	13	29 mm	8.8 mm	---
Building 3:					
Space between CIP column faces	7.187 m	50	59 mm	13.2 mm	*1.2%*
Space from CIP columns to exterior walls	4.779 m	7	28 mm	11.0	---
Building 4:					
Variations in column face dimensions about their means, ~450(*N*=13) & ~600 (*N*=41)mm	0.4537 m 0.6020 m	54	11 mm	2.0 mm	*4.3%*
Vertical distance from regression-adjusted reference line to underside of CIP slab.	0	38	17 mm	4.7 mm	42%
Horizontal distance from mullion face to edge of CIP slab	38.8 mm	61	22 mm	4.8 mm	14%
Horizontal distance from edge of CIP slab to regression line at mean slab-edge position	0	38	33 mm	6.5 mm	29%
Horizontal distance from mullion face to regression line at mean mullion position	0	62	11 mm	2.2 mm	15%
Mullion spacing	1.450 m	49	13 mm	2.4 mm	69%
Horizontal spaces between HSS suspenders	3.133 m	71	26 mm	4.9 mm	21%
Spaces between CIP columns, ~ 6 m	5.941	19	21 mm	5.0 mm	18%
Spaces between CIP columns, ~ 8.5 m	8.428 m	18	16 mm	4.4 mm	6.8%
Repetitions:					
Repeated measurements of an interval between two mullions, ~14 m	14.245 m	10	5 mm	1.5 mm	---

Dataset	***Mean***	***N***	***Range***	σ	χ^2
Repeated measurements of an interval between two mullions, ~33 m	33.444 m	11	50 mm	15.8 mm	---
Set of differences between paired measurements of mullion intervals ranging from 1.444 m to 29.945 m	0.2 mm	23	15 mm	3.6 mm	35%
Set of differences between paired measurements of a different set of mullion intervals ranging from 1.444 m to 47.702 m	-2.0 mm	37	14 mm	3.2 mm	21%
Combined set of differences between paired measurements of mullion intervals	-0.6 mm	60	17 mm	3.7 mm	72%

Table 1: Results of statistical analysis

MEAN VS. INTENDED POSITION

The mean sizes of CIP columns reveal a kind of error in construction that these measurements cannot address. For want of an option, one must assume that the mean size was the intended size, but it seems likely that for ~600 mm columns it was either 600 mm or 24 in., not the mean size of 602 mm, but which? Without knowing, and without extensive measurements of a larger number of buildings, there is no means of knowing what to expect of differences between mean constructed dimensions and intended dimensions. Because they vary so little, with these column sizes it is likely that a significant portion of the total variability is due to positioning and reading the steel tape.

OFFSETS

The measurements were often offsets between repeated elements, not their position relative to a fixed frame of reference, or known intended position. Assuming that

- there are no cumulative errors in layout, or more generally, that the location of each part is independent of the locations of other similar parts, and
- that the mean position is the intended position

it is possible to infer the standard deviation in absolute position from the standard deviation in offsets.[4] The standard deviation of a sum of *n* individual variables for each of which the standard deviation is known is

$$\sigma=\sqrt{(\sigma_1^2+\sigma_2^2\cdots+\sigma_n^2)} \qquad \text{Eqn. 1}$$

For pairs of identical objects (e.g. Columns, slab edges), where the individual standard deviations are equal, the standard deviation for the offset between them is thus

$$\sigma_o=\sqrt{(2\sigma_c^2)} \qquad \text{Eqn. 2}$$

where σ_o is the standard deviation of the offset and σ_c is the standard deviation for object position. So,

$$\sigma_c=\frac{\sigma_o}{\sqrt{2}} \qquad \text{Eqn. 3}$$

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[4] See Weisstein, *Convolution*

For columns, where offsets are measured between opposing faces, variation in column size would contribute to the variation in offsets, in addition to variation in the location of the column centres, so that

$$\sigma_o = \sqrt{(2(\sigma_c^2 + \sigma_f^2))} \qquad \text{Eqn. 4}$$

where σ_f is the standard deviation for column dimension in the direction of the offset. If $\sigma_c \gg \sigma_f$ then the square of the latter can be neglected, as has been done in Table 2. For an exterior wall, where adjoining columns would probably be laid out by locating the exterior face, it should also be left out, since all the variation in column size would occur at the inward face, making the values in Table 2, calculated from Eqn. 4, appropriate even if $\sigma_c \simeq \sigma_f$ or $\sigma_c \ll \sigma_f$.

Dataset	***Standard deviation (σ)***
Building 1:	
Horizontal position of CIP slab edges	5.2 mm
Building 2:	
North-to-South position of PC columns	14 mm
East-to-West position of PC columns (narrow set)	7.1 mm
Building 3:	
CIP column position	9.3 mm
Building 4:	
Horizontal position of HSS suspenders	1.7 mm
Position of CIP columns (spacing ~6 m)	3.5 mm
Position of CIP columns (spacing ~8.5 m)	3.1 mm

Table 2 Standard deviations in position determined from deviations in offsets.

CONCLUSIONS/RECOMMENDATIONS

The measurements made offer a starting point for working out tolerances for positioning of components when detailing for buildings similar to the four buildings measured.

There is not enough information here to fully inform designers, but it is recommended that the following steps should be taken:

- Recognize that all parts of a detail must either be positioned in some absolute frame of reference, with a degree of uncertainty, or cut-to-fit, or allowed to fall where they may as a result of contact or attachment to other parts.
- Provide for adjustable connections, space for errors to accumulate, or for cutting and fitting of elements that adjoin parts that are allowed to vary in position.
- Refer to information provided here, other similar sources, applicable standards, or educated guesses based on prior field experience to determine tolerances for each element, and then work out explicitly how adjustments will be made, where errors can be allowed to accumulate, and what can be cut-to-fit.
- Make tolerances explicit in the project documentation, even if the tolerances chosen are in line with applicable standards. Use absolute tolerances, knowing that outliers will occur, so that responsibility for cost of fixing them is clearly allocated.

In choosing tolerances to specify, and making allowance for them, it will be necessary to balance expected costs of providing for more adjustment, or otherwise coping with more variation, against expected reject rates. When deciding how much tolerance or adjustment to provide in a detail it is necessary to know (or guess) the variation to be expected and also to be able to anticipate the costs of remediation when actual construction falls out-of-bounds. Different reject rates may be tolerable, depending on cost of correcting out-of-tolerance construction. The positioning of structural members that support exterior cladding will need to be more accurate than the positioning of columns in an open area where ceilings and flooring can easily be cut-to-fit.

In the case of the cladding, allowing for a wider tolerance will reduce the cost of construction, but reduce rentable area, at least notionally (remembering that rents may finally be based on measurements made after construction). A simple, but conservative, approach would be to decide on a reject rate for each component dimension, determine a tolerance for each case from the standard deviation typical of that component dimension (based on data reported here or elsewhere), and sum the individual tolerances. Alternatively, one could calculate the cumulative tolerance:

$$T_c = \sqrt{(T_1^2 + T_2^2 \cdots + T_n^2)} \qquad \text{Eqn. 5}$$

This is valid if the causes of variation of individual components are unrelated, and if the selected tolerances are some multiple of the corresponding standard deviations. A more sophisticated approach is set out in ISO standards. Both approaches require standard deviations as input. All of these approaches assume that errors are random and Normally distributed, so actual reject rates will be higher than expected if misreading or cumulative errors in layout, or shifting in position in a consistent direction (e.g. deflection of CIP formwork) are allowed to occur unchecked.

Assuming that all errors are random, and Normally distributed, and that the standard deviation of the population (σ) is known, then the expected number of occurrences to fall within a given tolerance interval can be determined from Table 3.[5]

Acceptance rate	***Tolerance interval***
0.6826895	±1 σ
0.800	±1.28155 σ
0.900	±1.64485 σ
0.950	±1.95996 σ
0.9544997	±2 σ
0.990	±2.57583 σ
0.995	±2.80703 σ
0.9973002	±3 σ
0.999	±3.29053 σ

Table 3: Tolerances and corresponding rates of acceptance.

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[5] From Weisstein, *Standard Deviation*

Consider the standard deviations reported in Tables 1 and 2. Remember that on Building 1, for instance, if you were detailing a wall to be supported by the slab edges, and intended the detail to accommodate 99.5% of expected variation, you would need a tolerance of at least 5.2 mm x 2.807 on each side of the nominal position, i.e. ± 15 mm. Do the details you currently work with allow for that much adjustment?

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APPENDIX

For analysis, dropping some outlying values was tried to see if the remaining data would produce a satisfactory χ^2. Notes in Table 1 explain where this was done. This demonstrated (or reinforced, since Latta alludes to the effects of human error in measurement) that variation in a dimension can be expected to conform to the Normal distribution only if errors such as the transposition of numerals 3 and 8, misreading of instruments, or instrument positioning do not occur. For instance, initial analysis of the ~16 m column spacings in Building 2 showed outliers that were probably due to errors of this general kind:

- One value was probably misread. The smallest point might have been 16.685 instead of 15.585, but even making that change and using it still left this value by far the smallest.
- The second smallest value was still well below the next, and possibly didn't “belong” either.
- The largest value was also well above the next largest.

With these 3 values removed, the remaining data appeared to fit the Normal distribution well, but without the removal χ^2 is zero, and with only the one fairly clear recording error removed it is 0.6%. Without repeated measurement of the building, one cannot know which values “belonged”.

Similar errors undoubtedly occur during construction, and to extent that they do, variation in position may not conform to the expected distribution.

In the measurements of driving-aisle width between columns in Building 3, the two most negative of the 50 deviations from a mean value for aisle width of 7.187 m cause the frequency distribution fitting the histogram to be increasingly negatively skewed. Skewness calculated including both values was -1.02; dropping the most negative, -0.89; and dropping both, -0.77. Although these changes don't seem that large, the resulting improvements in the χ^2 tests at the 5% level, with 5 degrees of freedom, are from failing (1.2%), nearly passing (3.5%), to passing (7.3%). This was a common pattern for some of the failures and near-failures at the other buildings. On occasion, a physical explanation can be offered as justification for dropping the outliers (which seem always to be negative), but in this case, and some others, no such explanation could be identified.